Statistical methods for the detection of continuous gravitational waves

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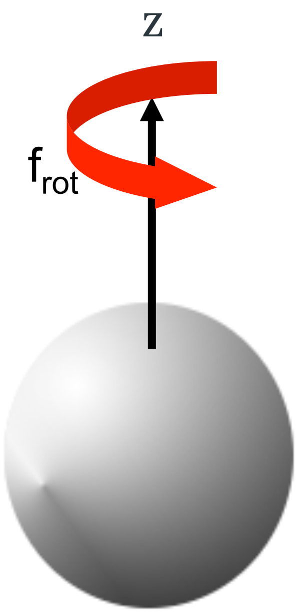
Deformation of a neutron star



ellipticity $\varepsilon = \frac{|I_{xx} - I_{yy}|}{I_{zz}}$

$f_{gw} = 2 f_{rot}$

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} \varepsilon f_{gw}^2}{D} = 3 \times 10^{-25}$$

for:

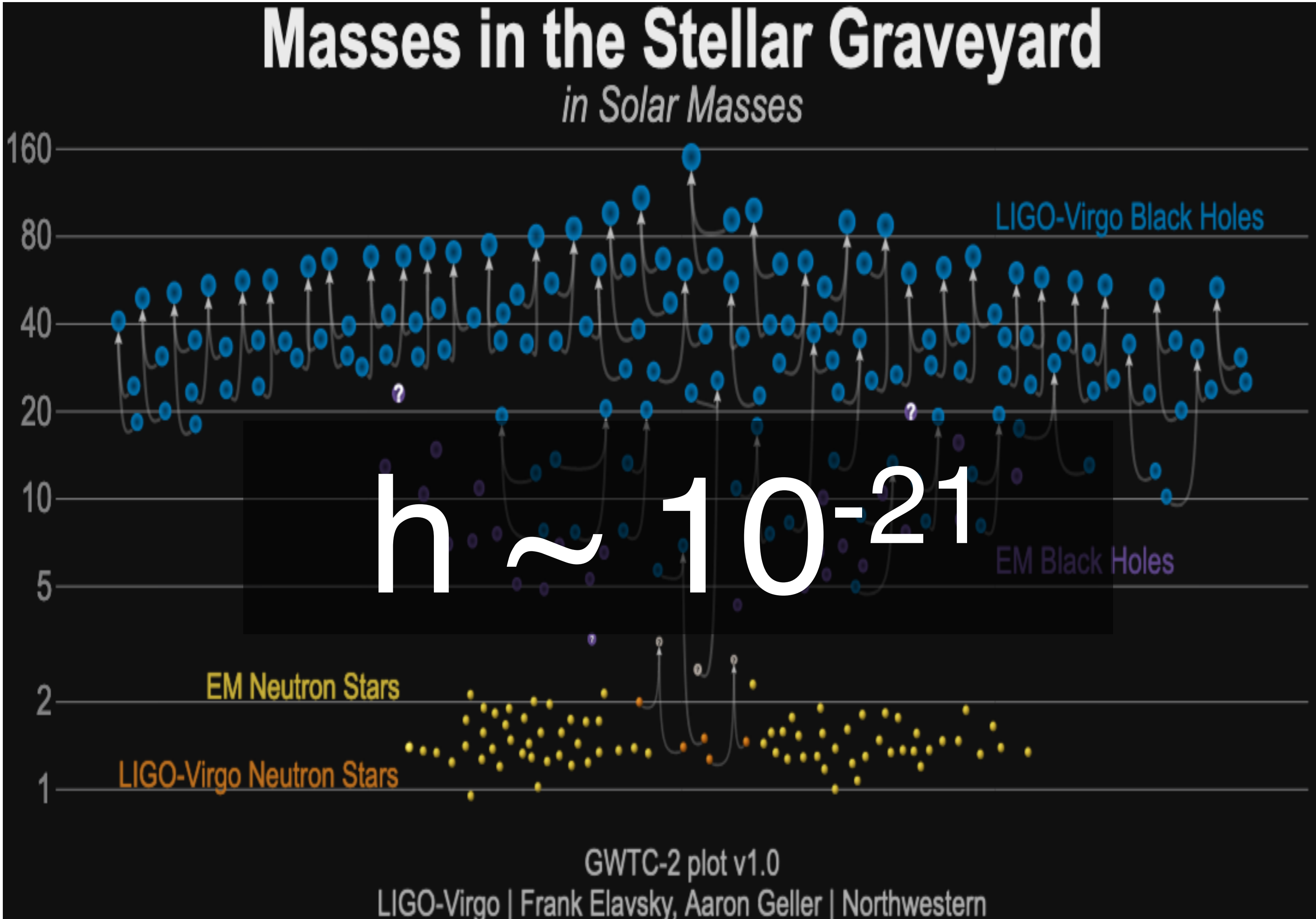
D=1kpc

f_{gw}=1kHz

$\varepsilon = 10^{-6}$

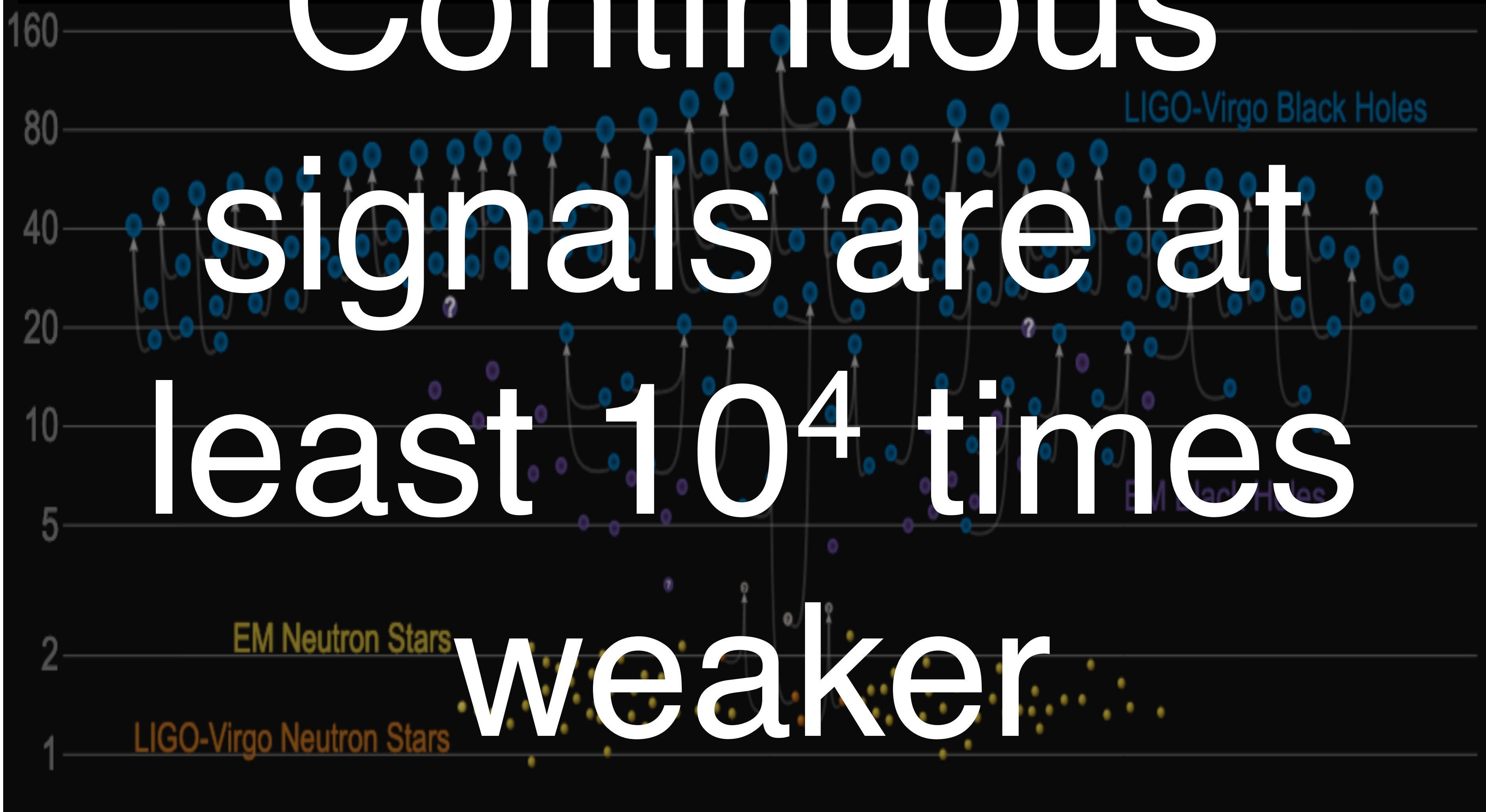
real value of ε ? Unknown.

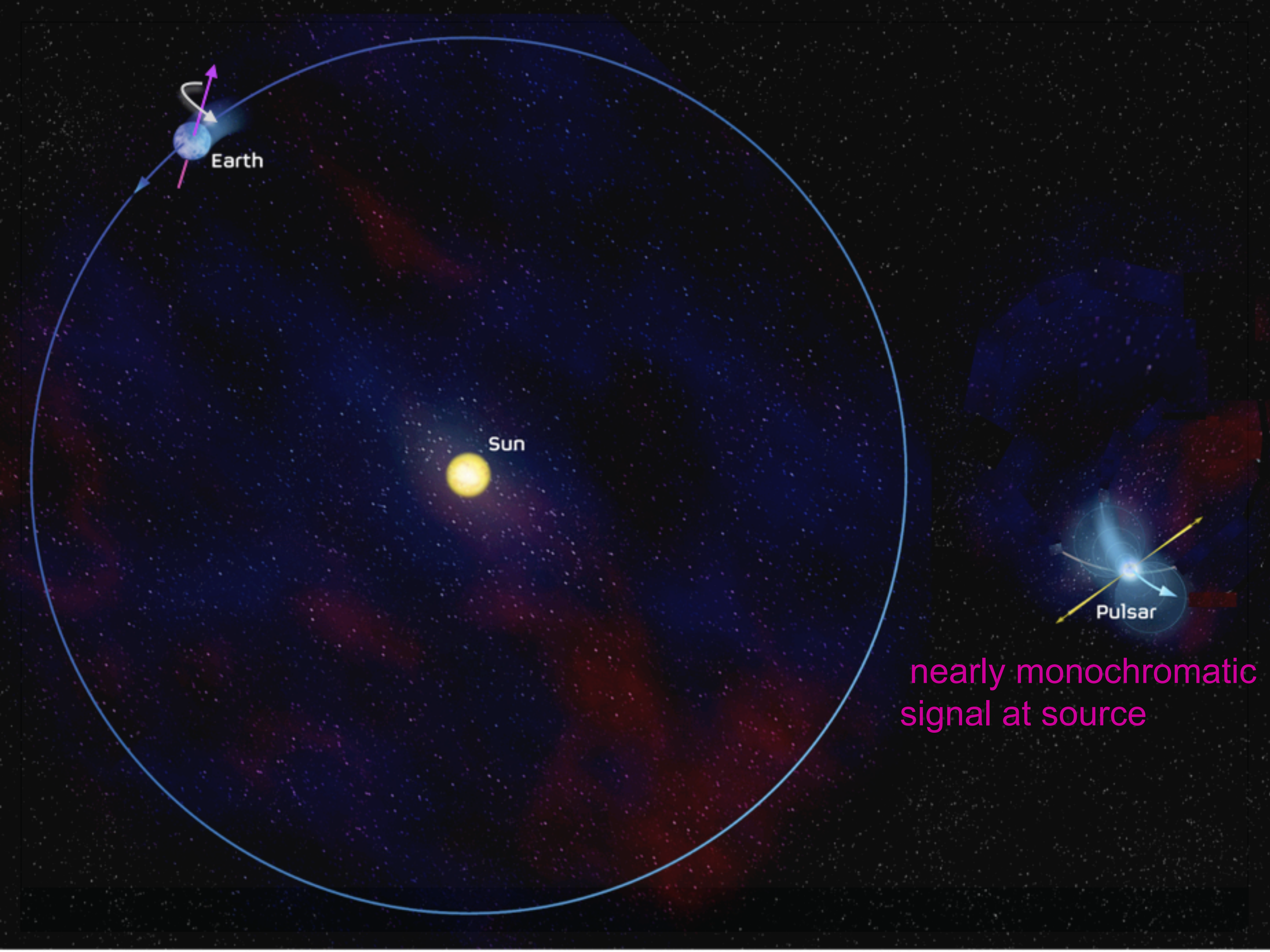
possible values: $10^{-12} - 10^{-5}$


Masses in the Stellar Graveyard
in Solar Masses
160
80
40
20
10
5
2
1
LIGO-Virgo Black Holes
EM Black Holes
EM Neutron Stars
LIGO-Virgo Neutron Stars
GWTC-2 plot v1.0
LIGO-Virgo | Frank Elavsky, Aaron Geller | Northwestern

$$h \sim 10^{-21}$$

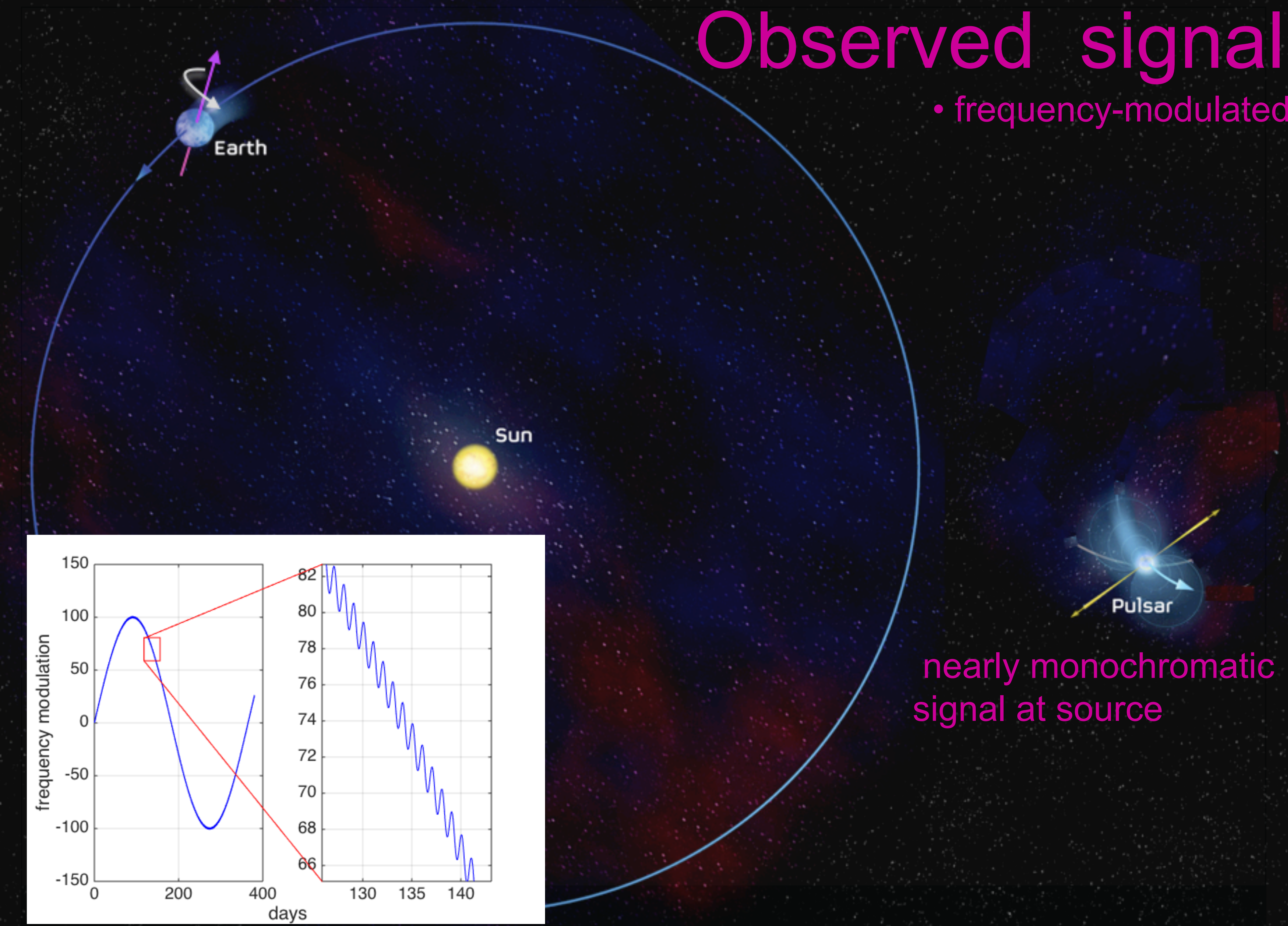
Continuous signals are at least 10^4 times weaker




Earth
Sun
Pulsar
nearly monochromatic
signal at source

Observed signal

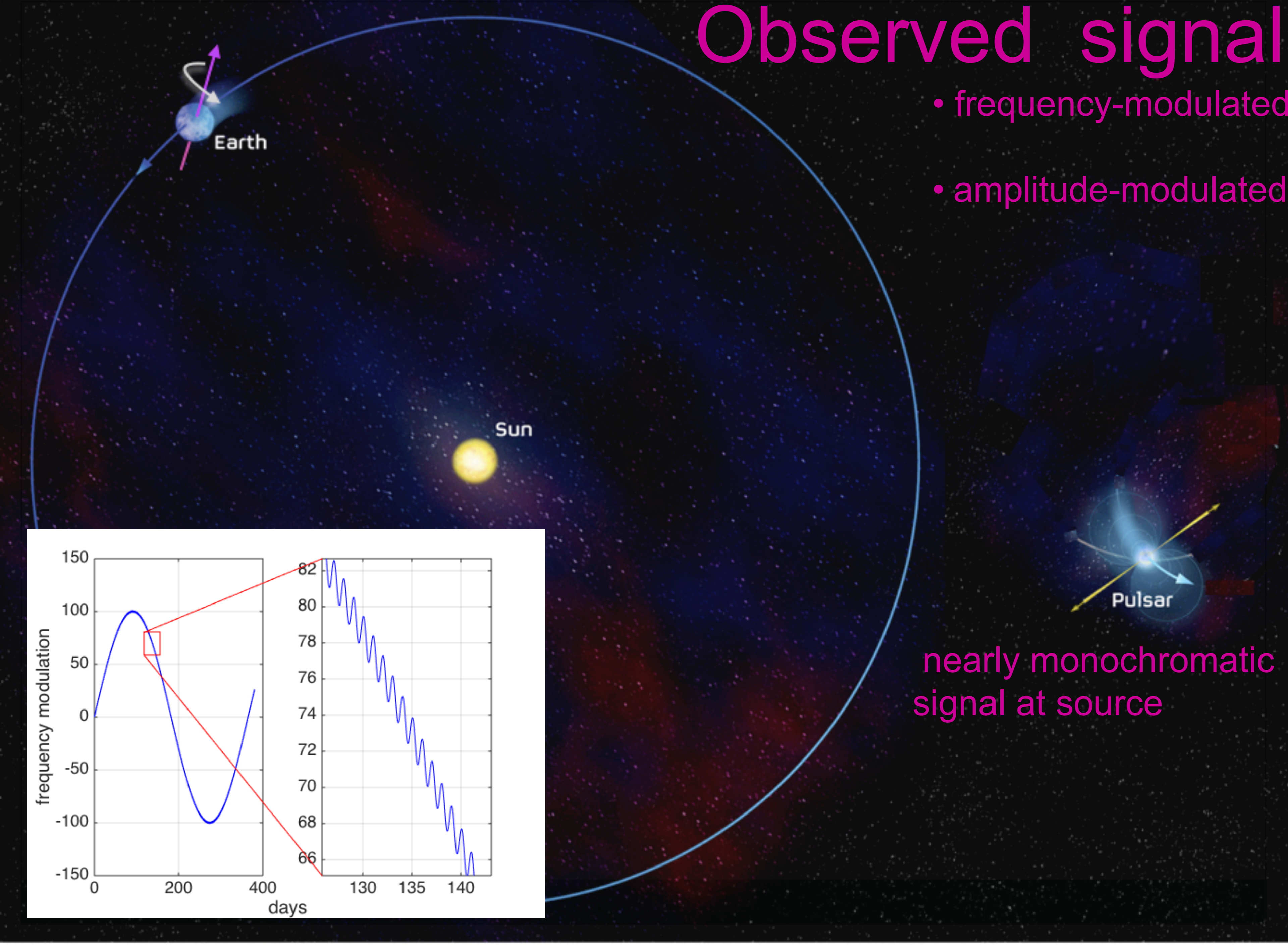
- frequency-modulated



nearly monochromatic signal at source

Observed signal

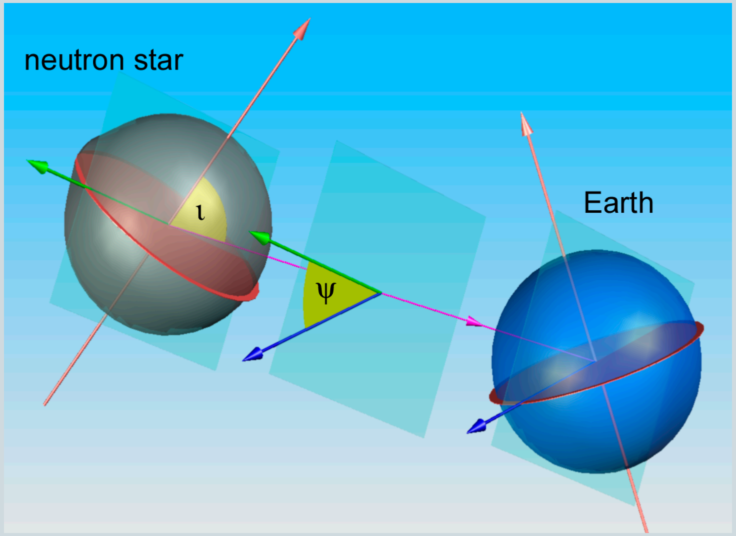
- frequency-modulated
- amplitude-modulated



nearly monochromatic signal at source

The signal-waveform parameters

- h_0 amplitude (distance, ellipticity)
- freq, freq derivatives, initial phase
- geometrical coupling factors:
  - ι
  - ψ



Coherent detection: frequency-domain methods

- "Correct" data to turn signal into a sinusoid
  - Frequency demodulation
  - Amplitude weighting according to antenna-sensitivity pattern
  - Inverse noise-weighting

- Take $|\mathrm{FFT}|^2$

  - F-statistic [1,2], 5-vector method [3], loosely coherent methods [4]

[1] Jaranowski et al, PRD 58 (1198), [2] Cutler&Schutz, PRD72 (2005), [3] Mastrogiovanni et al, CQG 34 (2017), [4] Dergachev arXiv1807:02351, [5] Dupuis&Woan, PRD 72 (2005)

Line-robust statistic

- F-statistic is the log-likelihood against Gaussian noise hypothesis, analytically maximized over cos ι, ψ and φ_0. Combines data from multiple detectors.
- But noise is not Gaussian, so:

Standard statistic

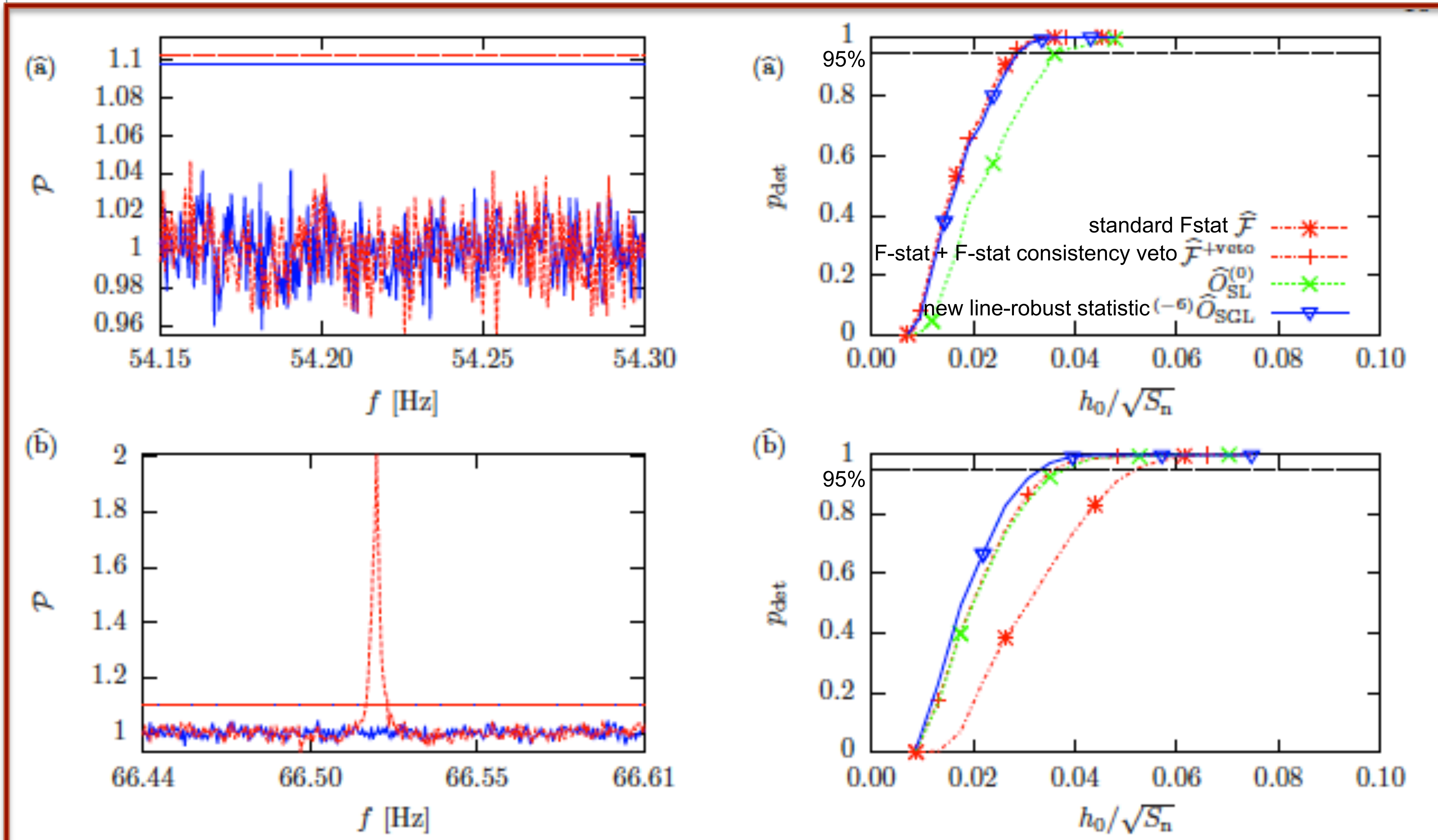
New statistic is an odds ratio

$$F = \frac{P(H_S|x)}{P(H_G|x)} \longrightarrow O_{SGL} = \frac{P(H_S|x)}{P(H_{GL}|x)}$$

- H_S is the signal + Gaussian-noise hypothesis
- H_{GL} is an expanded noise hypothesis : Gaussian noise *or* line-noise

D. Keitel, PRD 93, (2016) , D. Keitel et al , PRD 89, 2014

Performance in different noise conditions



Real detector data (noise): L1 in red, H1 in blue

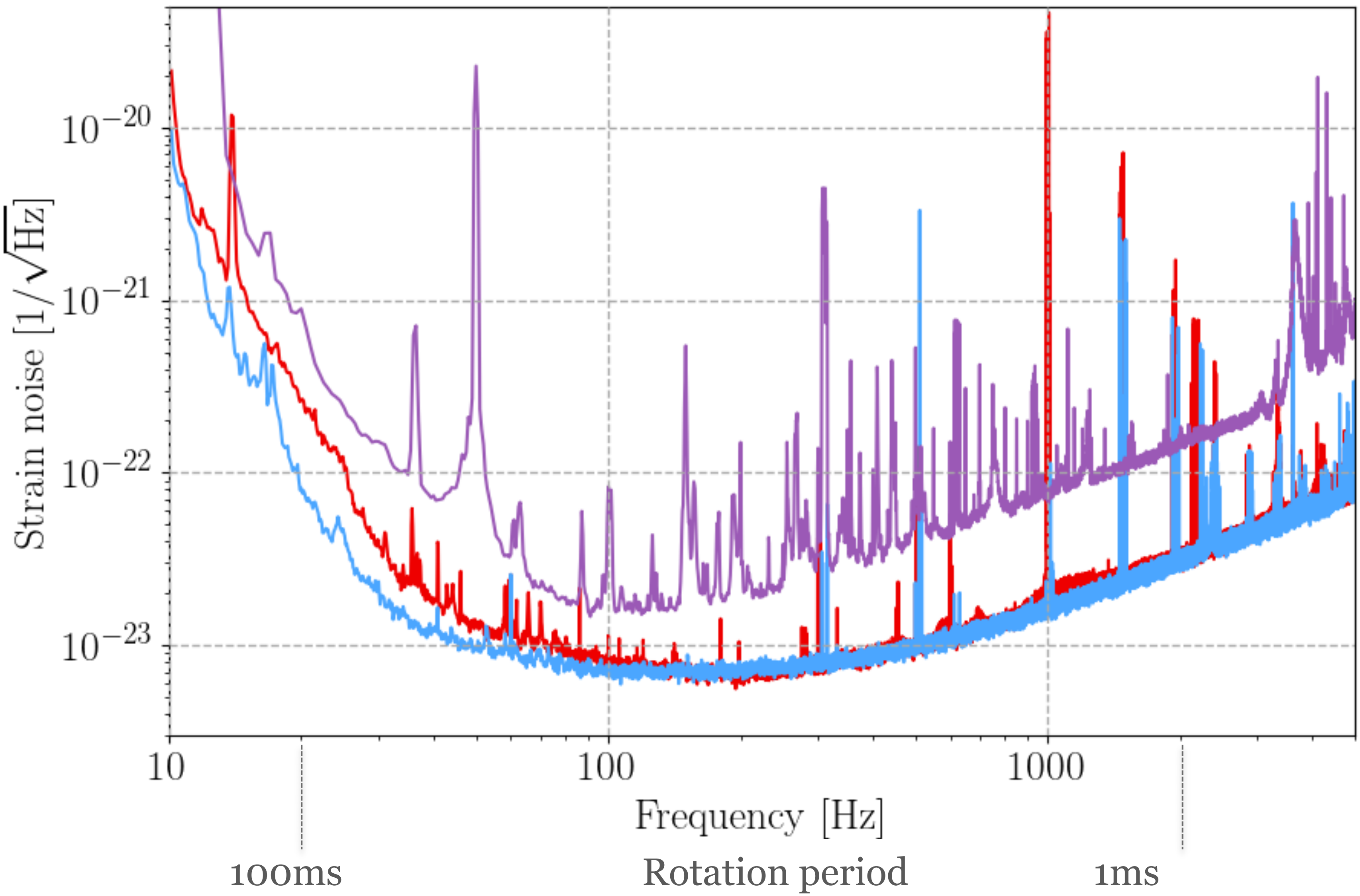
Detection probability for injected signals of different amplitudes in that noise.

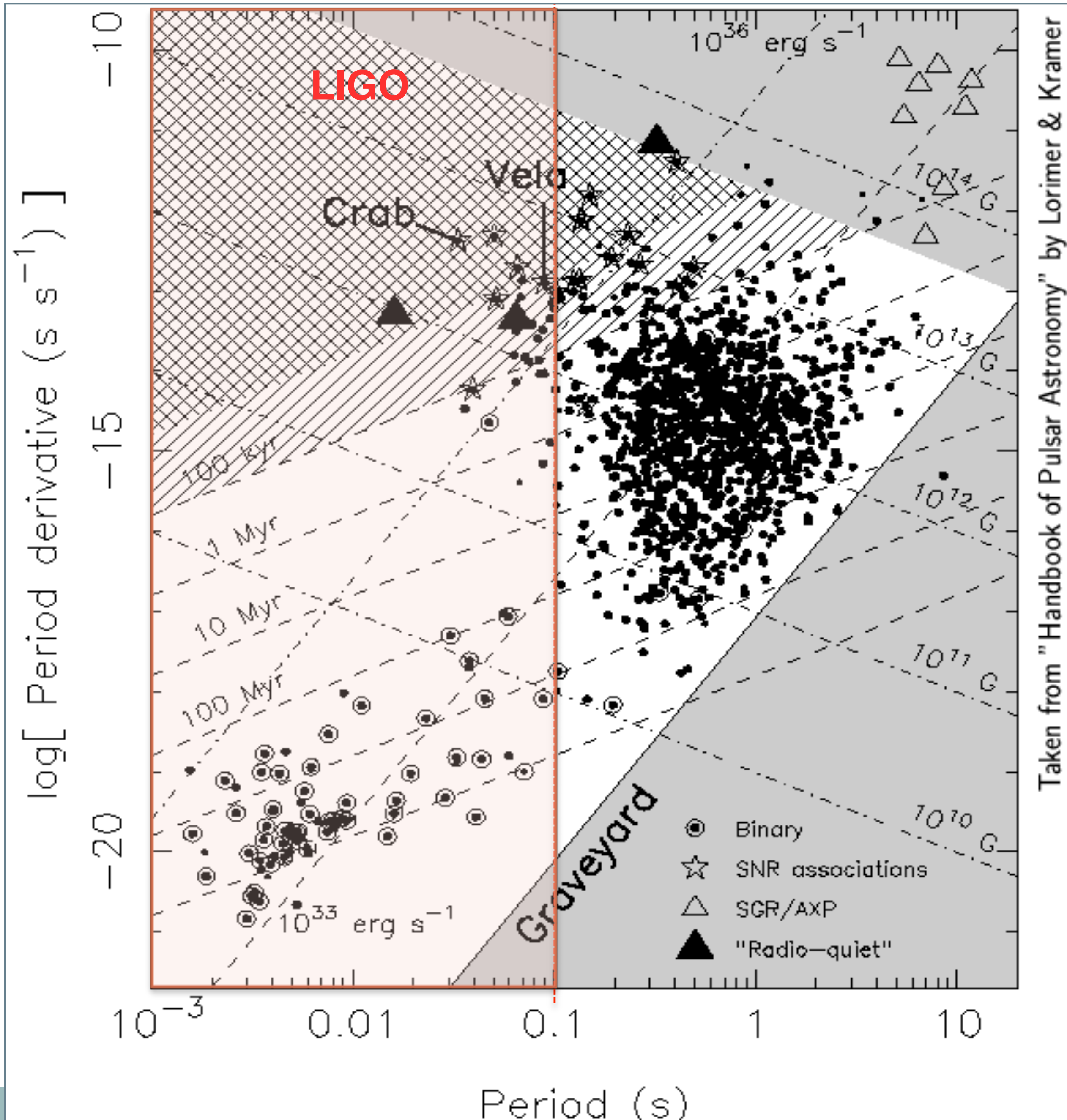
Coherent detection: time-domain methods

- Two stages
  - Frequency de-modulation + heterodyning and low-pass filtering (band pass and down-sample)
  - Parameter estimation, construction posterior
    - Set upper limits
    - Model selection
- Mostly used for searches for emission from known pulsars

Dupuis&Woan, PRD 72 (2005), Pitkin et al, arXiv:1603.00412 (2012), Pitkin et al arXiv:1705.08978 (2017), Pitkin et al, PRD 98 (2018)

GW detectors' noise




LIGO
Crab
Vela
10^{36} erg s^{-1}
10^{33} erg s^{-1}
10^{14} G
10^{13} G
10^{12} G
10^{11} G
10^{10} G
100 kyr
1 Myr
10 Myr
100 Myr
Graveyard
Binary
SNR associations
SGR/AXP
"Radio—quiet"
−10
−15
−20
10^{-3}
0.01
0.1
1
10
log[Period derivative (s s^{-1})]
Period (s)
Taken from "Handbook of Pulsar Astronomy" by Lorimer & Kramer

Bayesian

- Posterior probability of a given signal s, given the data {x} :

$$p(s \mid \{x\}) \propto p(s) \cdot p(\{x\} \mid s)$$

posterior prob on signal

prior

prob of data given signal

Bayesian posteriors

- Posterior on amplitude: marginalize over the unknown/uncertain parameters $\phi_0,\psi,\cos\iota$

$$p(h_0|\{x\}) = \iiint p(\{x\}| h_0,\varphi_0,\psi,\cos i) \text{ x}$$

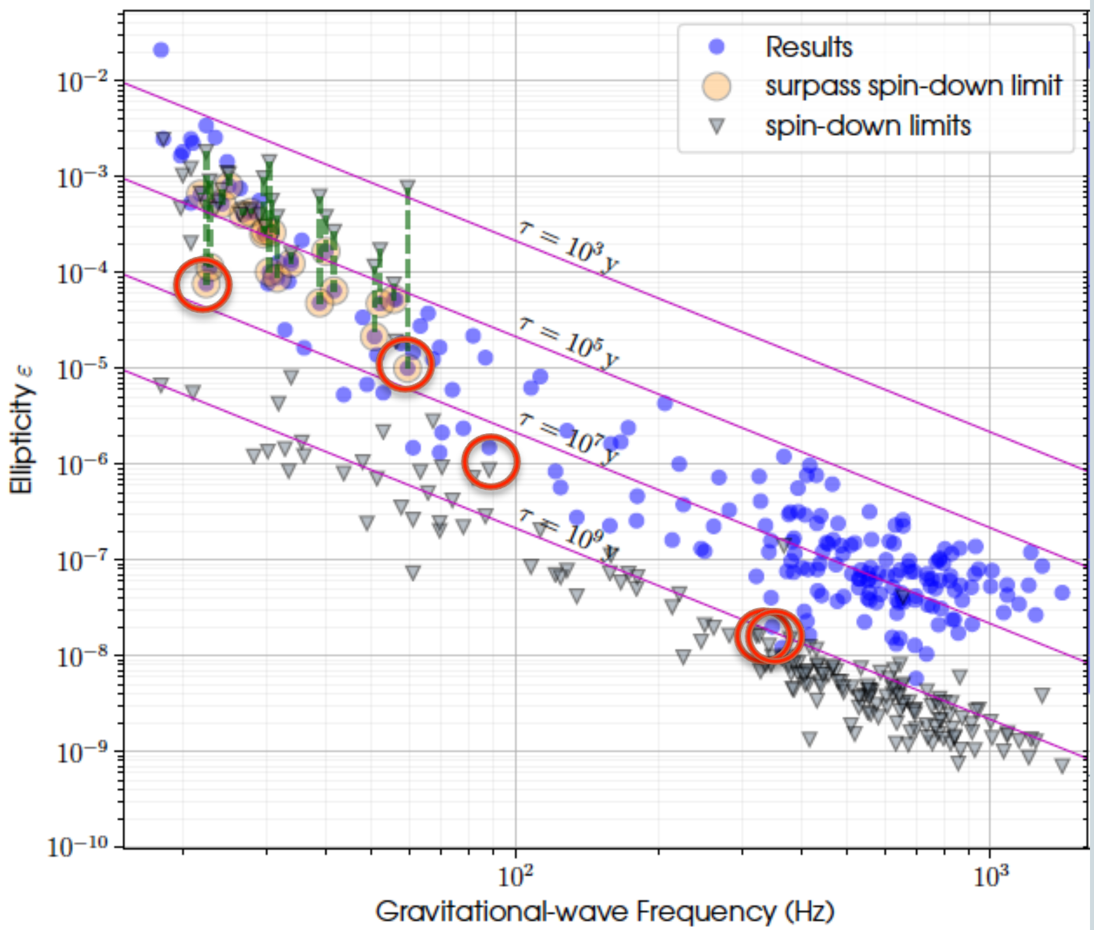
$$\text{x } p(\varphi_0)d\varphi_0 \, p(\psi)d\psi \, p(\cos i)d\cos i$$

- Upper limit: integrate to the required total probability (confidence level) and read-off the corresponding h_0 upper limit value

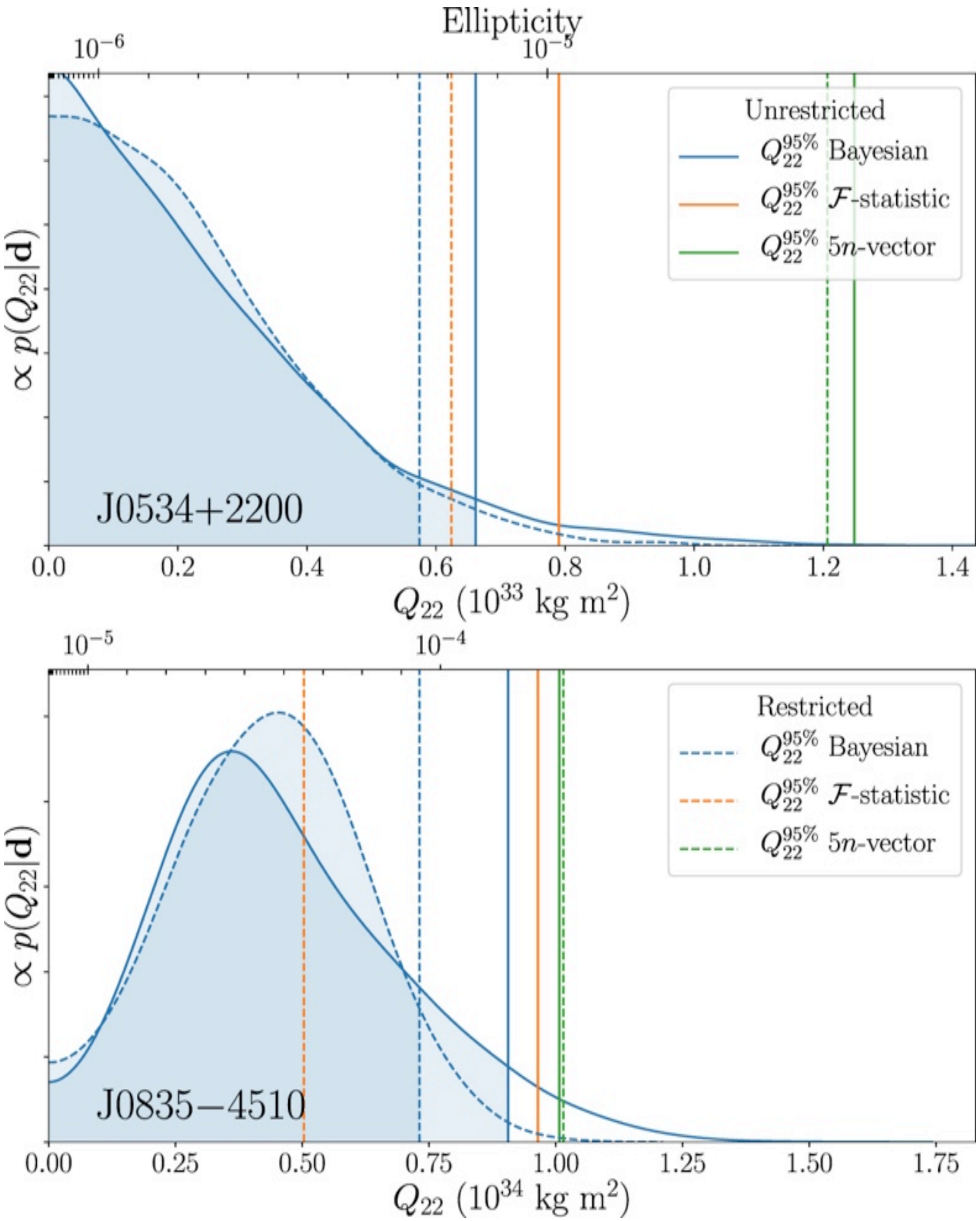
- Translate into upper limit on deformation: $h_0 = \frac{4\pi^2 G}{c^4}\frac{I_{zz}\varepsilon f_{gw}^2}{D}$

➔ new LIGO results on 5 pulsars

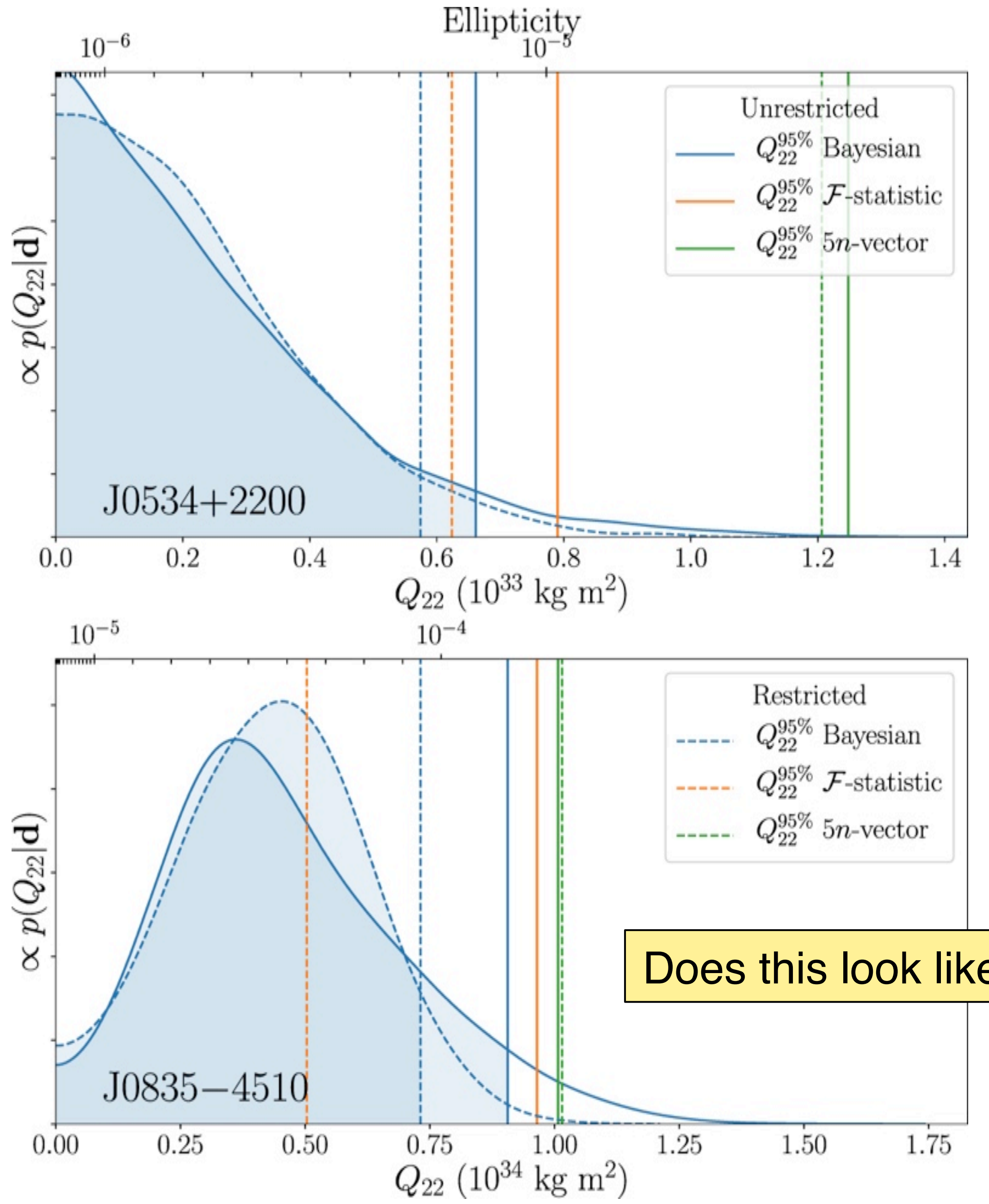
(ApJL 902, L21, 2020)



- J0437–4715, 347.4 Hz, jus below spindown limit
- J0711–6830, 364.2 Hz, @70% of spindown limit
- J0737–3039A 88.2 Hz, @ ≈spindown limit
- Crab (59.2 Hz) @1% of spindown limit + Vela (22.4 Hz) @7% of spindown limit



LVC, ApJ Lett 902, L21 (2020)

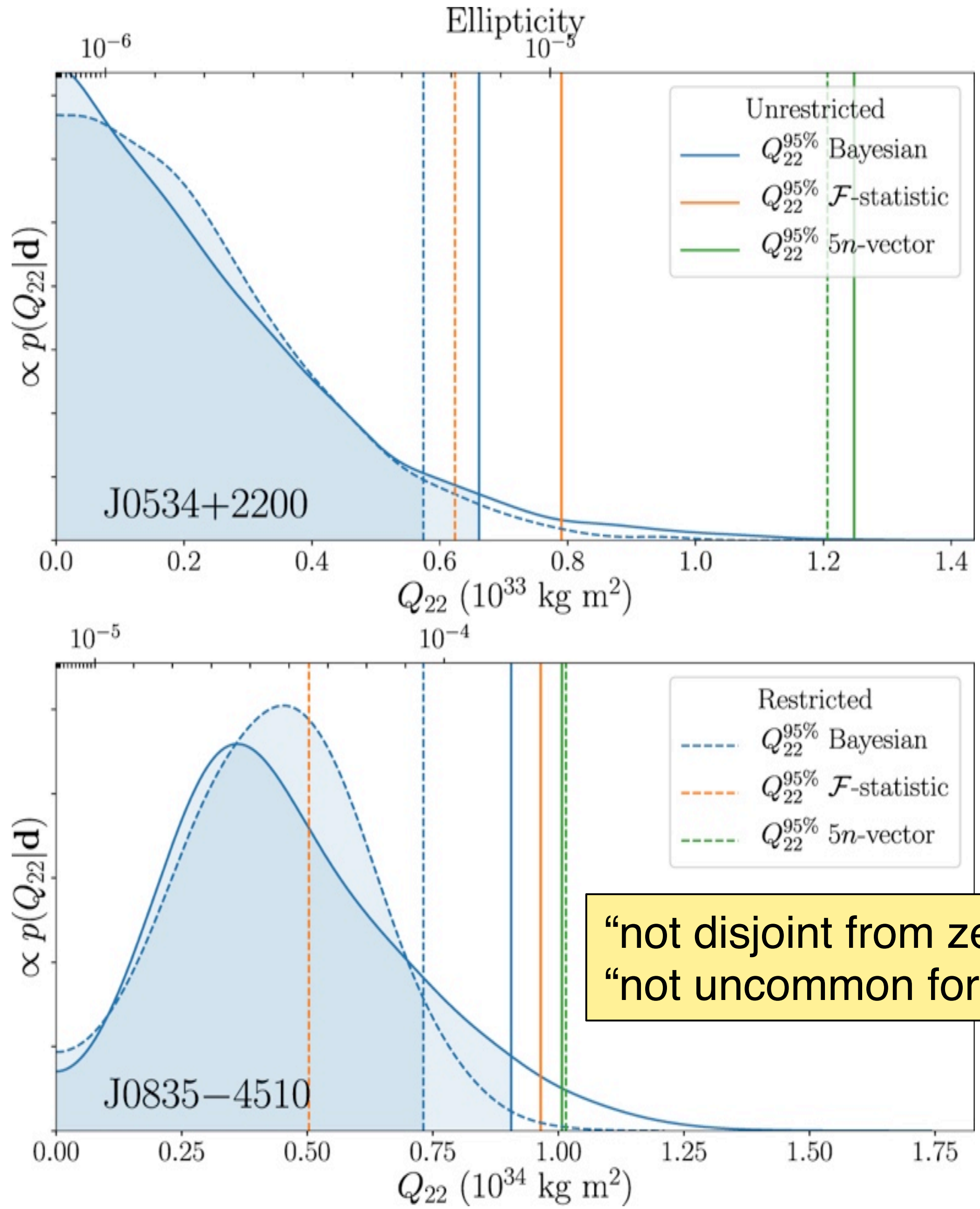


Does this look like a signal ?

LVC, ApJ Lett 902, L21 (2020)

Establishing detection confidence

- **would it be significant in Gaussian noise ?**
- can we exclude a noise disturbance (instrumental/environmental) in the data causing such result ?
- Does the result stay significant if we evaluate it against search results from real detector noise ?
  - Estimating the background



"not disjoint from zero"
"not uncommon for pure Gaussian noise"

Establishing detection confidence

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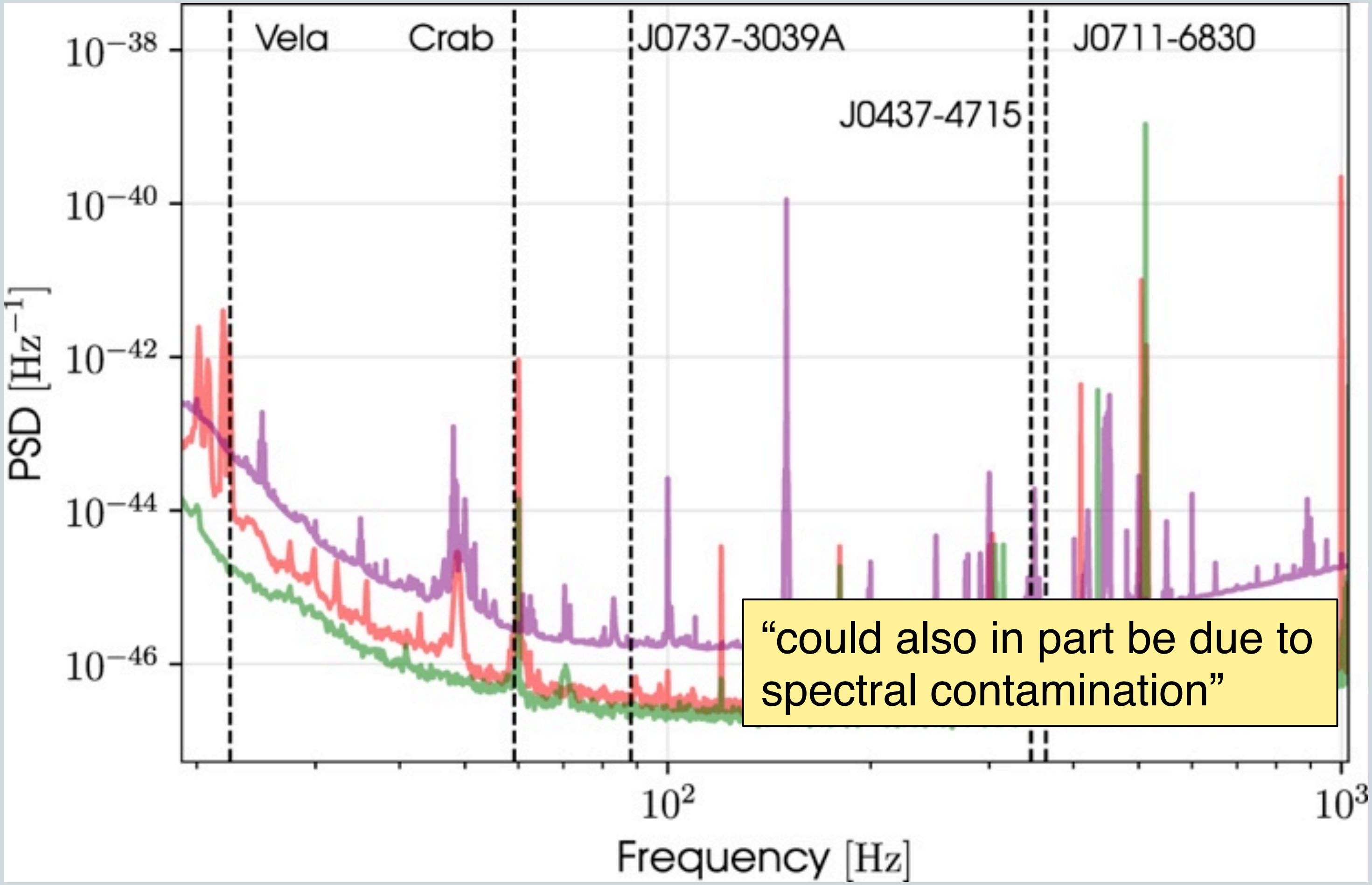
Establishing detection confidence



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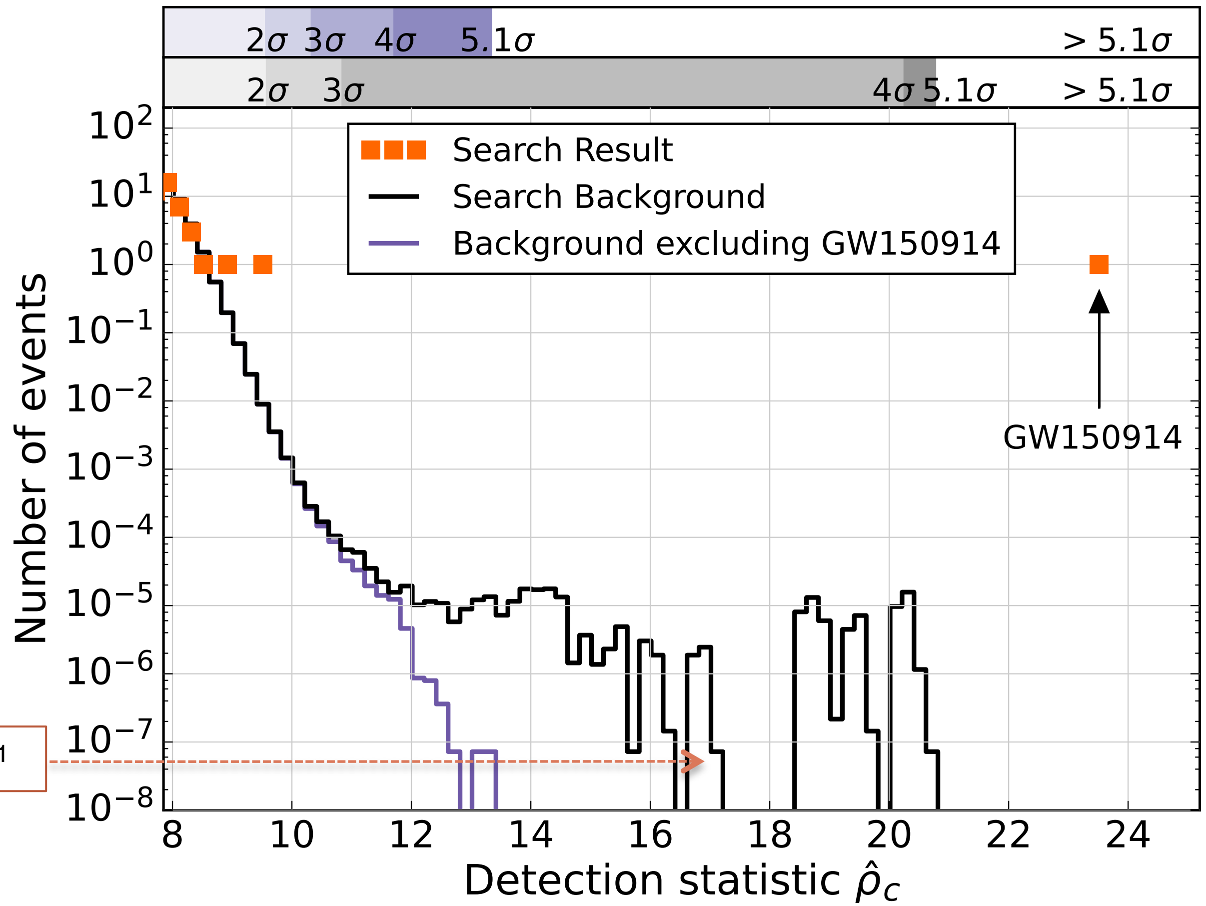
  - **Estimating the background**

The first GW detection

[Observation of Gravitational Waves from a Binary Black Hole Merger](#)

Phys.Rev.Lett. 116 (2016)

1.4 x 10^7 time slides corresponding to 608 000 yrs of simulated background.

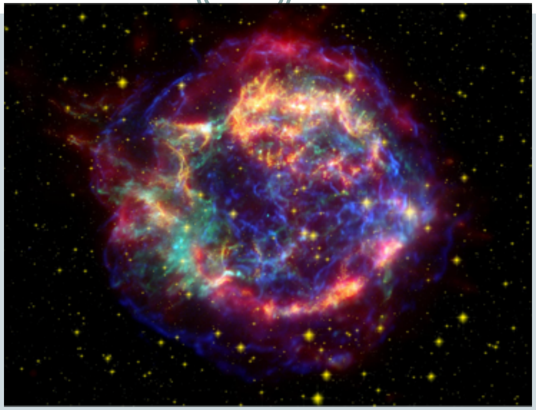


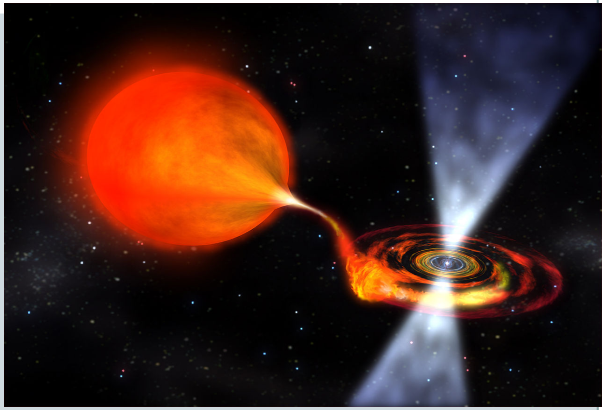
$7\times10^{-8} \approx (1.4 \times 10^7)^{-1}$

Establishing detection confidence

- For a search for emission from a known pulsar it should be possible to estimate the background:
  - Repeating the same search many times "off-source"
    - near-by frequencies (extensive literature)
    - different sky positions, Isi et al, arXiv:2010.12612 (2020)
- Not so simple for other types of continuous wave searches

Broad searches



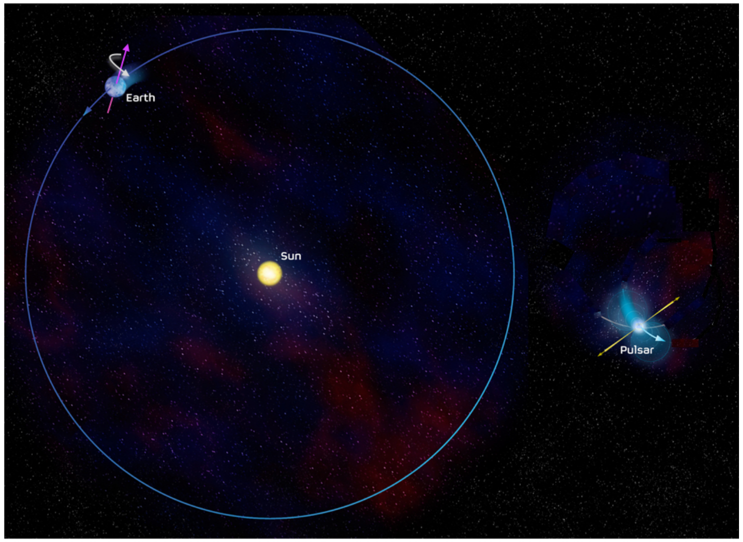


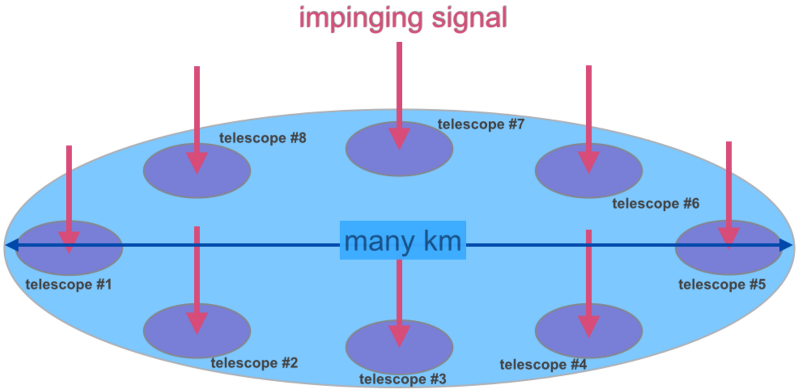
Interesting regions (Galactic center)

Interesting objects (e.g. CasA or the Neutron star in ScoX-1)

All-sky

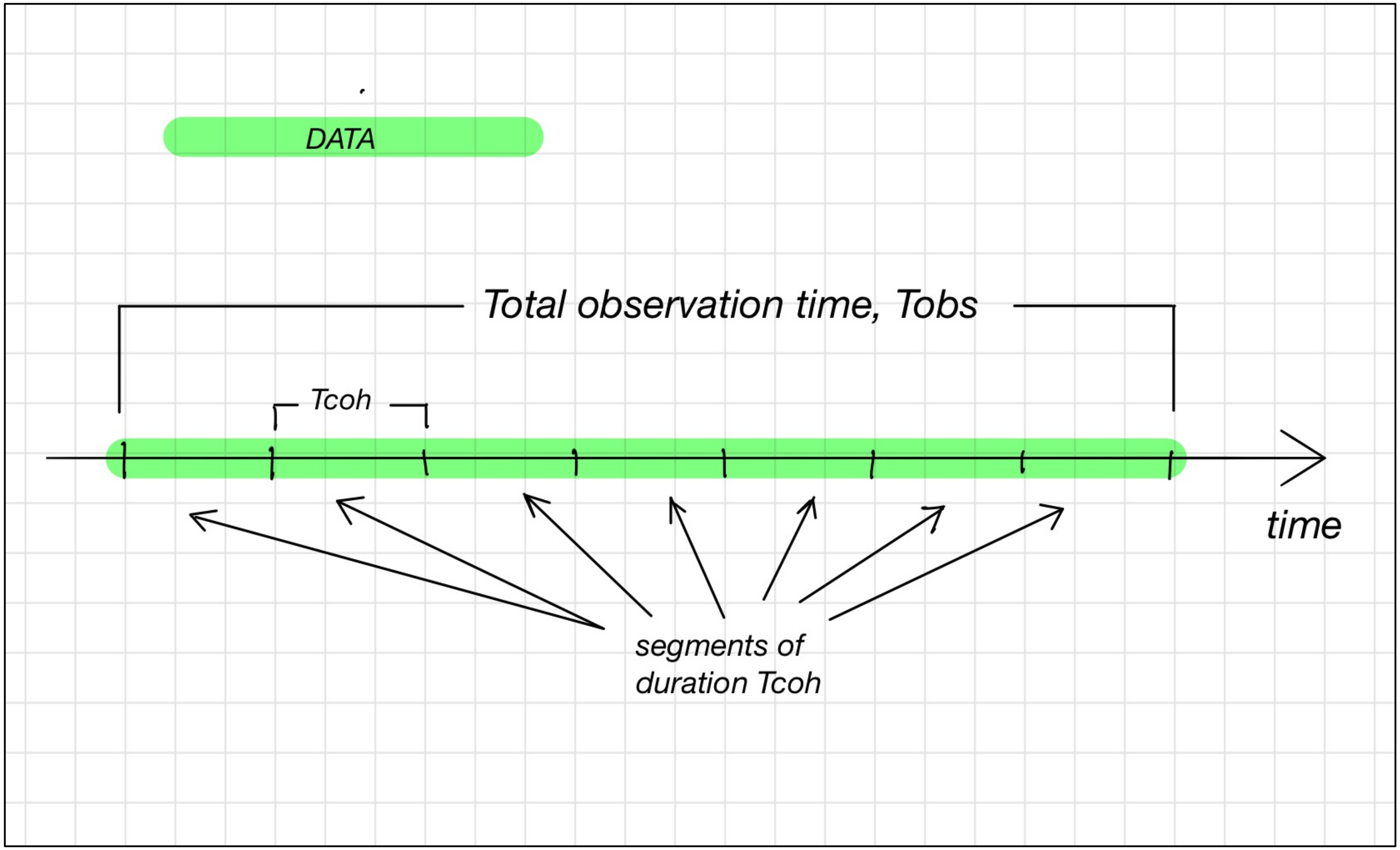
Long coherent observations make for too expensive searches





- like aperture synthesis for radio telescopes
- the baseline in this case is the diameter of the Earth's orbit around the Sun, hence yielding resolutions < 4 arcsec (@100Hz)

Semi-coherent detection methods



Brady et al, PRD 57 (1998), Brady&Creighton, PRD 61 (2000), Dhurandhar et al, PRD 77 (2008), Walsh et al, PRD 94 (2016), O. Piccinni et al, CQG 36 (2019), Dergachev&Papa, PRL 123 (2019)

Hierarchical schemes

A cascade of semi-coherent searches. At each stage:

- Tcoh increases
- more noise is rejected
- the SNR of a signal-candidate increases
- the uncertainty in the signal parameters decreases

Very complex

Search	T_{coh} hr	N_{seg}	δf μHz	$\delta \dot{f}$ 10^{-14} Hz/s	m_{sky}	$< \mu >$	Δf μHz	$\Delta \dot{f}$ 10^{-14} Hz/s	$\frac{r_{sky}}{d(8.0 \times 10^{-3})}$	R^a	N_{in}	N_{out}
Stage 0	60	64	3.34	32.747 9	8.0×10^{-3}	0.5	full range	full range	all-sky	–	7.9×10^{17}	350 145
Stage 1	60	64	3.34	20	5.0×10^{-4}	0.3	850.0	1.2×10^{-10}	5.0	0.75	350 145	101 001
Stage 2	126	29	1	2	1.0×10^{-5}	0.09	130.0	2.0×10^{-11}	0.75	1.99	101 001	11 915
Stage 3	126	29	0.19	2	1.0×10^{-7}	0.002	10.0	2.0×10^{-12}	0.1	2.2	11 915	6 128
Stage 4	250	14	0.025	2	2.5×10^{-8}	0.001	0.4	3.2×10^{-13}	0.02	4.3	6 128	33
Stage 5	500	7	0.01	1	1.0×10^{-8}	0.001	0.17	1.45×10^{-13}	0.008	6.0	33	21
Stage 6	1 000	2	0.001	0.1	1.0×10^{-9}	0.000 2	0.067	6.4×10^{-14}	0.003 7	10.0	21	18
Stage 7	1 563	2	0.001	0.1	5.0×10^{-10}	0.000 1	0.05	8.0×10^{-14}	0.005	15.0	18	8
Stage 8	≈ 5 486	1	0.001	0.1	1.0×10^{-10}	0.000 7	0.032 5	4.25×10^{-14}	0.002 5	50.0	8	6

Steltner et al,, arXiv:2009.12260, to appear in ApJ (2020)

Assessing significance in right out of broad parameter search

- **very hard on original search**
- **emerging strategy: assess significance of a simpler, “verification search”**
  - **independent data**
  - **fewer templates**

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Assessing significance in broad parameter searches

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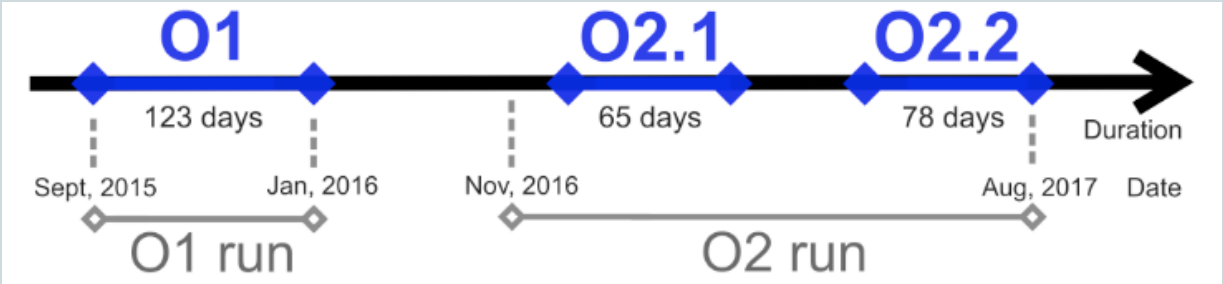
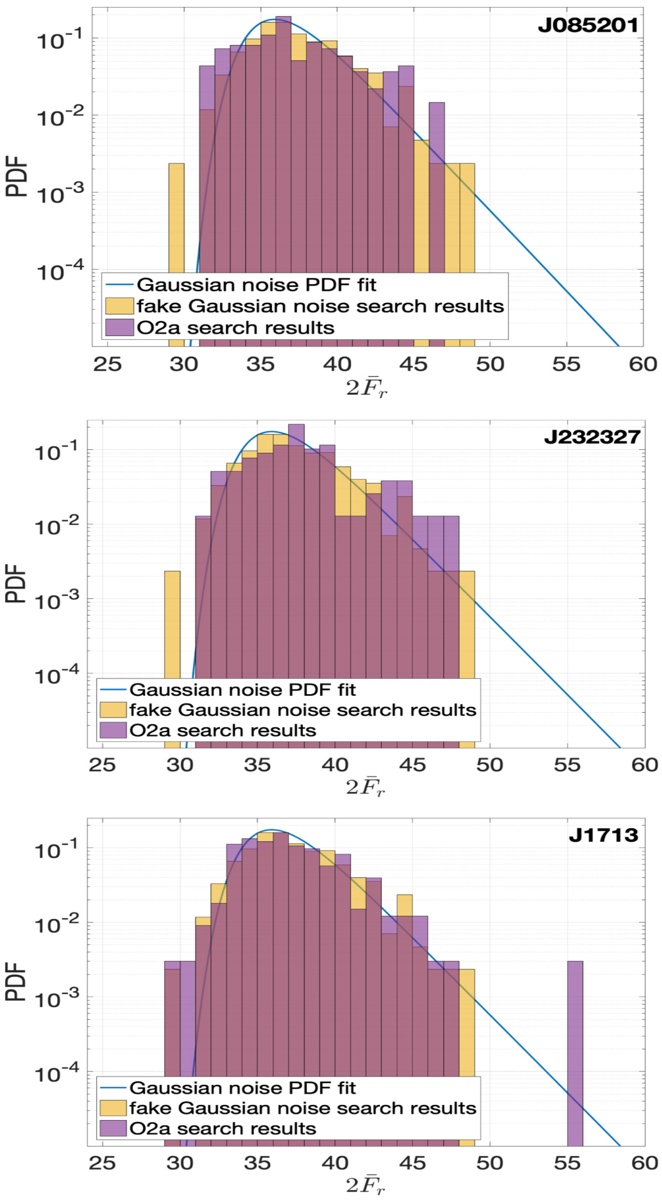


Ming et al, PRD 100 (2019); Papa et al, Astrophys.J. 89 (2020)

- O1 search:
  - 2×10^{17} waveforms searched
  - surviving 575

O2.1 search results

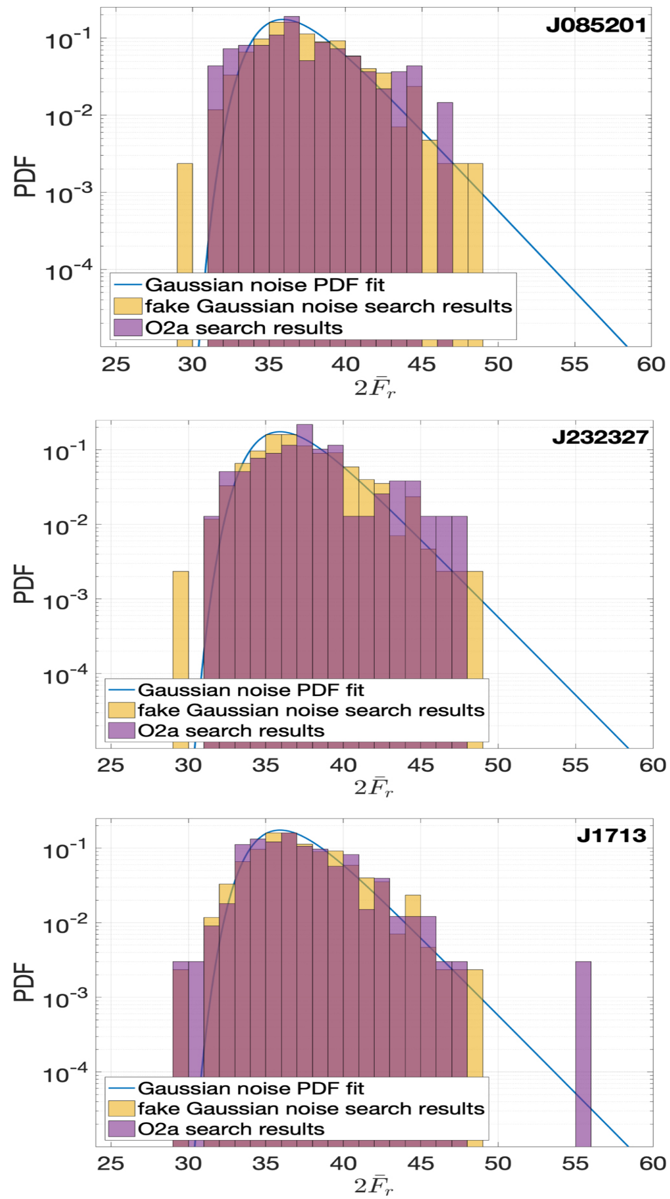
- O1 search:
  - 2×10^{17} waveforms searched
  - surviving 575
- O2.1 search:
  - surviving 1



Papa et al, Astrophys.J. 897 (2020) 1, 22

O2.1 search results

- O1 search:
  - 2×10^{17} waveforms searched
  - surviving 575
- O2.1 search:
  - surviving 1
- O2.2 search:
  - not confirmed
- extensive x-ray search on archival data
  - not confirmed
- turned out not to be a gold-plated candidate



Common predicament ?

- Some searches have no surviving outliers:
  - Lindblom&Owen, PRD 101, (2020)
  - Millhouse et al, PRD 102 (2020)
  - Covas&Sintes, PRL 124 (2020)
  - Steltner et al, to appear in ApJ, arXiv:2009.12260 (2020)
  - Zhang et al, arXiv:2011.04414 (2020)

- Others produce outliers that survive all automated thresholds and checks but are not completely convincing and need verification on new data

  - "None of these searches has found clear evidence for a CW signal [..] The remaining 26 sub-threshold candidates, which will be further analyzed in a forthcoming work", Abbott at al, PRD100 (2019)

  - "The search yields a number of low-significance, above threshold candidates [that…] will be followed up in subsequent observing runs.", Middleton et al, PRD 102 (2020)

  - "No significant associated signal is identified […] A focused gravitational-wave search in O3 data based on the parameters provided here should be easily able to shed light..", Papa et al, ApJ897 (2020)

  - "We list outliers […] Targeted searches [on O3 data] based on the information presented here […] should be straightforward. ". Dergachev&Papa, PRL125 (2020)

Concluding remarks: known pulsar searches

- in spite of efforts continuous gravitational waves still elude detection

- the assessment of the significance of a signal *from a pulsar* will be relatively easy
  - Several proven detection schemes exist
  - Well-established collaboration between LVC and pulsar astronomers
  - Machinery is in place for construction of posteriors and model selection

Concluding remarks: broad surveys

- Different situation for broad surveys
- A first detection á la GW150914, appears to me increasingly unlikely
  - more likely is a marginal candidate, with evidence building up over different GW data sets or/and through the identification of an electromagnetic counterpart.
    - assessment of significance is all but trivial, not mature
      - assessment of instrumental artefacts, time-critical
      - folding-in EM follow-up results
  - contemplate possibility signal may deviate from assumptions
    - need to push sensitivity of robust methods, with shorter coherence lengths
    - the sensitivity assessment is even trickier

THANK YOU !